The Utility of Case Study Methodology in Mathematics Teacher Preparation

By Azita Manouchehri & Mary C. Enderson

Recent mathematics education reform in the United States proposes that teachers of mathematics should act as facilitators of learning in the classroom and promote robust mathematical thinking among students. They must strive to create learning environments focused on genuine inquiry and mathematical discourse (NCTM 1991, 2000). Implementing this type of teaching is demanding and complex. In addition to possessing a detailed understanding of mathematics, teachers must have a sophisticated knowledge of how children learn the subject (Ball 1993). Moreover, they must be sensitive to the complexities of classroom interactions (Yackel & Cobb 1996). Teachers need to be capable of analyzing student learning in the presence of specific teaching actions and curriculum materials (McClain & Cobb 2001). In light of these demands, mathematics teacher education is faced with the challenge of orienting future teachers towards these new methods of teaching while simultaneously assisting them acquire new mathematical understandings, and helping them develop the analytical skills

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they need to initiate and sustain inquiry about practice. The path to meeting this challenge is neither smooth nor straightforward. A major difficulty lies within the domain of current understandings and attitudes of future teachers.

Research has established that many prospective teachers enter their teacher education programs with the perception that teaching is a simple task (Brown & Borko 1992). Their optimism towards their ability to teach serves as a barrier to their serious engagement in teacher education and in analyzing practice (Thompson 1992). In light of these results, recommendations towards reform in teacher education has called for defining ways in which teachers' beliefs and thinking about the adequacy of their knowledge base for teaching is challenged, and for finding alternative ways of problematizing practice for them (Cooney 1994). As a means to meet this goal, Shulman (1992) encouraged teacher educators to use cases studies in the course of pedagogical preparation of future teachers. Other researchers have also recommended using case study methodology as a vehicle to motivate teachers' critical reflection upon professional issues (Broudy 1990; Walen & Williams, 2000; Hutchines 1993; Kagan 1993; Schon 1991; McWilliam 1992).

Supporters of the use of this strategy in teacher preparation have argued that the benefit of using such environments is at least twofold. On the one hand, these cases grant prospective teachers the opportunity to become familiar with the problems that arise in real classroom settings and prepare them for what they would encounter later as they begin to teach (Skyes & Bird 1993). On the other hand, they award teachers some background information about the setting they will experience (Merseth 1996; Carnegie Forum on Education 1986). Other advantages of using case studies include pedagogical problem solving opportunities (Walen & Williams 2000), development of interpersonal and communication skills (Barnette & Tyson 1993), and simultaneous training of content areas (Barnette 1991; Wilson 1992). While literature on the use of case-study method in teacher education is scarce (Elksin 1998), several researchers have recommended it in various educational arenas (Hallahan & Kauffman 1994; Wasserman 1994; Richardson 1996; Skyes 1989; McWilliam 1992). Richert (1992) sums up the need for careful study of this strategy in teacher preparation:

> The development of case methods for teacher education must be accompanied by a research agenda that seeks to illuminate what prospective teachers actually learn, and do not learn, from different genres of cases and the instructional methods that best support this learning… Our optimism for the power of case methods in teacher education must be bolstered by our understanding of the underlying questions related to teaching and learning with cases, as well as to issues concerned with learning to teach. (pp.237-238)

The need to explore the potential of case study strategy on teacher development motivated the current research. In this work, we evaluated the impact of the case study methodology on 50 secondary mathematics teacher candidates enrolled in

Methods of Teaching Mathematics courses we taught. Two questions focused the data collection process:

1. Does the use of case study method increase the participants' understanding of the complexities associated with reform-based teaching and learning mathematics?

2. Does the use of case study method increase the participants' discourse about mathematics and teaching?

Theoretical Considerations

Our work is grounded in a constructivist view of learning (Cobb & Bausersfeld 1995) and a Vygotskiian (1978, 1986) view of teaching. We perceive individuals as meaning makers who use conceptual structures to interpret what they see, hear, or read. We view teaching as (a) assisting students to build conceptual methods that are useful for learning mathematical and pedagogical concepts, and (b) helping students to develop more sophisticated thinking strategies as they become able to do so. This assistance takes the form of designing problem contexts that can enable them to construct the knowledge they need for a more advanced analysis of concepts.

This combination of views leads to a conception of teaching and learning as assisting the performance of learners with their own meaning making. Therefore, we believe teaching requires continual learning about the knowing state of students that are both short- and long-term based. Our Vygotskian view of teaching leads us to careful selection of activities that might help individuals develop the mental structures they need to engage in deeper analysis of mathematical and pedagogical concepts they study. We believe it necessary to design and utilize tasks that create dissonance in students' current understandings, thus, preparing them to deal with problems in a more sophisticated manner.

We also believe, as Vygotsky explained, that a person's meaning and actions are social/cultural constructions, and it follows that we can only understand the individual within the content of social relations in which the individual exists (Wertsch 1986). Grounded in this theory is the understanding that successful members of a society must know and understand the established local orders-the specific roles, rights, and responsibilities that the social system establishes and enforces. For the adoption of a public meaning of knowledge to be viable to the individual, it must be the product of social understandings. To be a successful member of a society, a person needs to understand and gain access to social norms and social constructs that work within that society. Because individual sense making, or the formation of meaning, is intrinsically, inevitably, and profoundly social, each individual comes to know what it means to hold a certain role in the context of social interactions. Although each individual brings her own constructs and meanings to the situation, it, in turn, interactively influences the making of meaning and sets the standard for cultural competence (Danziger 1990).

In light of this we believe that future teachers learn about the cultural norms of the profession through dialogue and interaction with their colleagues and peers. Moreover, these interactions help them establish what is worthwhile knowledge and what is valued for practice. We argue, based on the perspective of social constructivist theory, that by facilitating communication among the future teachers, and by orchestrating situations in which they must exchange ideas, articulate their thinking, and attempt to solve conflicting views, they develop the capacity to see new perspectives and build new understandings about mathematics and teaching. This framework guides not only the type of experiences we provide our students in class but also how we define our role in the process of educating them. We explicitly inform the students that our goal is to challenge their analysis and to be a "critical friend" during their discourse. In this way, we strive to create a community of learners in which students rely on feedback from peers and work towards developing reasoning skills based on collective inquiry.

Context and Content: Mathematics Methods Course

At both our institutions the methods-of-teaching-mathematics course is taken at the end of the future teachers' program of study. Upon completing the methods course, they begin their student teaching phase. Commonly the students enrolled in the methods course have completed all their required mathematics coursework. Among our student population are those with an equivalent of an undergraduate degree in mathematics, who are seeking secondary teaching certification. Some of our students may be enrolled in the undergraduate program in secondary mathematics education, and there may be some post baccalaureate students and graduate students pursuing a master's in mathematics education.

Five features characterize our students' backgrounds: (1) Their knowledge of mathematics is, for the most part, procedural and algorithmic; (2) they have limited knowledge of instructional tools and techniques useful for teaching mathematics beyond chalkboard presentations and lectures; (3) they have limited experience with mathematical problem solving as learners; (4) they have had a practicum experience (4-6 weeks) in a school setting during which they assisted teachers, worked with individual or small groups of students, and wrote and taught between 2 to 3 lessons. Upon completion of the methods course, students enter their student teaching phase.

Research Plan

For the purpose of collecting data for the current research we designed uniform experiences for participants at both sites to minimize variability of conditions that could affect the reliability of results. We also organized an implementation timeline along with the sequence in which the activities were to be used at both sites.

We used case studies of examples and non-examples of reform-based instruction to engage the participants in reflective analysis of mathematics, teaching, learning, and curriculum. In deciding case studies, we considered three specific criteria. These are described below.

1. We wanted each case to address an important concept in 9-12 curriculum. It was our intention to use each case as a vehicle to engage the participants in the study of mathematics from a conceptual perspective.

2. We wanted the case to present a dilemma. It had to provide a context for examination of specific teaching tools and representations, thus, facilitating their inquiry in pedagogical content knowledge.

3. We wanted the case to be open-ended, motivating the generation of "what if" questions that demanded further study. In this way we wanted to force the participants to go beyond the given story, try to formulate hypotheses and later verify them by relying on findings from current research.

Thus, we chose nine cases to be used with participants. These cases were based on real classroom events, each portraying a teaching or learning dilemma (See Appendix for examples).. We presented each case to the students and asked them to respond to the following items in groups.

◆ Analyze the episode from the point of view of the learners. What are the dilemmas? How are their needs accommodated or denied?

◆ Analyze the episode from the point of view of the teacher. What theory is guiding her instruction?

◆ Analyze the episode from the point of view of the Professional Standards for Teaching Mathematics. Which standards are met? Which standards are not met?

◆ Analyze the episode from the point of view of the Curriculum Standards. Which of the Standards are met?

◆ If you were the teacher, what would you have done differently? Why?

Each week a whole-group discussion followed the small-group analysis of the case we presented in class. During these discussions we encouraged the participants to share their analyses of the case they studied, to assess the various perspectives presented by their peers (to agree or disagree with each other), and to suggest additional questions that needed to be explored.[1]

Data Collection and Analysis

During the first two weeks of the course (2 class sessions/6 hours), we introduced the participants to the National Council of Teachers of Mathematics (NCTM) Principles and Standards for School Mathematics (2000), and Professional Standards for Teaching Mathematics (1991). As they read the various sections from each book, we asked the participants to analyze and assess their own mathematics education as learners in light of those standards, and to rate their understanding and level of comfort with the type of teaching proposed by both documents. Moreover, we insisted that they would identify those areas that appeared vague and ambiguous to them.

Students were asked to write and submit a reflective journal at the end of the second week of the class. They were instructed to provide specific information on the following three topics.

1. Their reactions to the NCTM's standards—whether they found the recommendations for teaching, learning, and curriculum easy or difficult to implement.

2. Their particular beliefs about how students learn mathematics, and the type of teaching that best fosters student learning.

3. The most pressing challenges they felt they would face in the course of their career as a teacher.

The participants' responses were examined to prepare a profile of their conceptions about teaching and learning and their reactions to the Standards upon entering the course. These responses were studied later to identify any changes in their thinking as the result of their exposure to case analysis activities. The same assignment was required at the end of the 13th week of the course. The students were asked to write a second journal entry responding to the same questions listed above. A comparison of pre- and post-intervention responses then followed.

Once a week, for a period of 8 weeks, we provided the students with a specific case. Participants were instructed to first discuss each case in small groups of 3 to 4. A whole group discussion followed the small-group analysis. During the whole-group discussions, group members shared their analysis with their peers in class. Whole-group discussions were videotaped and later transcribed. We intended to study the anatomy and content of discourse among the participants during their case analysis sessions; therefore, data was analyzed to determine dominant themes of discourse among them. Analysis began with cataloging video-segments, and examining field notes to provide evidence related to:

The kinds of discussions that the participants had.

The issues they raised in the course of their discussions of the cases.

Their foci of attention during case analysis episodes.

Each of us studied the videotapes and the participants' journals in her respective setting first and identified the trends and themes that emerged. Once the initial categorization of data was completed, a cross examination of findings at both sites was conducted. We searched for similarities and differences among the data at both sites. These themes were used to draw conclusions about the impact of the use of case study method in both settings.

In order to establish reliability of the results, each of us selected a random sample of video-clips from her setting to be reviewed and analyzed by the second collaborating author. Our notes on the same video-clips were then compared to assure a match between our interpretations of the participants' actions and talk. Promising clips from both settings then were selected to serve as evidence in our final preparation of the research report.

Findings

The results of the study indicate that case studies created situations in which discussions about learning, learners, teaching, and curriculum emerged naturally and in contexts meaningful to the participants. In small and large groups, the participants explored both the mathematical ideas that were presented in each case and evaluated the learners' and the teachers' work. The data indicates that these explorations influenced the participants' assessment of professional issues, including the challenges they anticipated facing and their own ability to implement reformed-based curriculum and instruction. In what follows we report (a) findings from the initial journal entries the participants submitted and (b) findings that emerged from our intervention study. The initial data are presented first in order to provide a profile of the knowledge base and the attitudes the participants brought to our classes. We will use these results to further analyze the impact of the use of case studies on the participants' developing perspectives and knowledge base.

Initial Data

Table 1 summarizes the participants' responses to the three focused questions we had assigned them at the beginning of the study.

In the initial journal entry a majority of the participants (n=45) expressed that they found the NCTM's Standards for Curriculum and Teaching to be sensible and easy to implement. Five participants were unsure of the content of the Curriculum Standards for more advanced high school courses. They also suggested that they did not fully understand how the sequence of mathematical concepts recommended for high school curriculum translated into specific grade levels.[2]

All 50 participants found the Professional Standards for Teaching Mathematics compatible with their own thinking about how mathematics should be taught. Moreover, 46 participants believed they were capable of implementing these new visions of teaching. These results were consistent with the findings of previous studies highlighting the future teacher's optimism toward their ability to teach (Pape 1992; Pajaros 1991).

Table 1
Summary of Results from the First Journal Entry

Themes	Site 1 (n=27)	Site 2 (n=23)	Total (n=50)
Analysis of Standards			
Content Standards are trivial- I fully understand them	25	20	45
Not sure how to implement them in specific grade levels	3	2	5
Professional Teaching Standards are easy to implement— They are in line with my own thinking	27	23	50
I know how to implement the teaching envisioned	27	23	50
Analysis of self			
Confident about teaching	25	23	48
Not sure if I can bring down the content to the kids' level	7		7
Concerns and Challenges			
Managing students	27	23	50
Students lack of interest in learning	27	23	50
Students' inadequate mathematics background	20	23	43
Students learn best when teachers have/do			
Clear explanations	20	18	38
Sequence the content from simple to hard	25	23	48
Relate to kids	21	20	41
Enthusiasm and energy	20	15	35
Patience	20	22	42
Caring	18	19	37
Use enough examples	17	15	32

The professional challenges that 48 of the participants anticipated included: (1) classroom management and (2) their ability to succeed with students who lacked motivation and for whom learning mathematics was not a priority. This group expressed that they expected the methods course would provide them with a series of "fun" activities they could use to motivate student participation in class. Forty participants also identified the challenge of working with those students who lacked basic skills and whose mathematical background was less than adequate to learn high school mathematics. These teachers also hoped that the methods course would provide them with concrete examples of how to deal with this dilemma in class.

All 50 participants believed mathematics is learned best if the teacher explains ideas clearly, shows enough examples to make concepts understood, and uses contexts that are meaningful to learners. They all articulated the belief that a teacher has to sequence the lessons carefully to assure coverage of ideas from simple to more complex. Further, they viewed this procedure as essential to building confidence among learners. The participants believed that by leaving gaps during the lecture, not demonstrating enough examples of how to solve certain problems, and not breaking ideas into manageable pieces, the teacher risks losing students. Forty participants believed that the excitement that the teacher showed for what she taught also affected whether students learned mathematics.

All participants identified having "patience," "ability to relate to students' lives," and "skills in explaining concepts in a simple language so that students understood them" to be some of the most important attributes of a mathematics teacher. All participants believed they possessed these qualities, with some qualification. Five of the participants thought they might encounter difficulty in "talking mathematics in kid language."

The participants' initial insights and responses allowed us to frame their intellectual and analytical standing within the perspective provided by previous research. As reported in literature, our research participants also seemed to possess a degree of optimism in regard to their ability to teach. They felt confident in influencing student learning and viewed the teacher as the key source for knowledge development among students. Moreover, all of the participants seemed most concerned about the affective aspect of classroom interaction rather than the cognitive development of students. They also viewed the primary value of the methods course in providing them with "interesting" and "fun" activities they could use to overcome the learning problems in the classroom. Neither student learning nor curriculum was problematic to them. Such was the case even for those who claimed little knowledge about the content of high school mathematics curriculum. It seemed that the assigned readings on the reform documents neither created dissonance in their thinking about teaching and curriculum, nor motivated them to reflect on their own understanding of mathematics. This became an increasingly important point of analysis later in the study as we tried to evaluate the impact of using case studies on their thinking.

Reactions to Case Studies

The participants differed on what they saw as problematic in each case we presented to them and what they identified as appropriate teaching actions in each scenario. These differences motivated their engagement in professional discourse and led them to a more reflective critical evaluation of teaching issues. This, however, was not an immediate consequence of the use of case studies in groups. In reporting the participants' reactions and what they learned from these experiences we present three phases that served as benchmarks in our progression.

Weeks 1-2: Detachment and low level of involvement. During the first two weeks of implementing the case study activities, the participants appeared reluctant to seriously engage in analysis of vignettes presented to them. While only a handful of them were concerned about the mathematical theme of each case, a majority of participants focused more on the teacher's actions and her interactions with the students. The participants were concerned about whether the teacher had attended to the emotional needs of her students. During the first two weeks none of the participants paid attention to students' work and thinking as presented in the vignettes. Not one of the participants tested the mathematical accuracy of what students had suggested in the case. Moreover, none of the participants inquired additional information about the setting that was portrayed in each episode.

In their analysis of the Fraction case (Case I), for instance, almost all the participants noted they would have insisted that the students do more examples using the standard algorithm of "inverting and multiplying." They also stated that although they would praise the student for having generated a different method for dividing fractions, they would ask everyone in class to test the accuracy of the algorithm for other fractions. It must be noted that none of the participants even tried to determine the legitimacy of the student-generated algorithm by testing it. In the same case, all participants commended the teacher for having used manipulative materials in her lesson to allow students to explore various relationships. It appeared that the mathematical outcome of the session was not particularly important to them. Moreover, interpreting the situation from the point of view of the learners was particularly difficult for them. Three participants indicated some concern for the short- and long-term impact of the encounter on the student portrayed in the episode. Relying more on personal experiences, they were worried that the child might have felt left out from the rest of the class and may have become uninterested in doing mathematics as result. In response to the guiding questions we had provided them, all of the participants had difficulty articulating the connections between the content of the Curriculum Standards (2000) and the mathematics that was discussed in the episode. None of the students seemed able to frame their analysis by the guidelines set by NCTM's standards for content and pedagogy (NCTM 1991, 2000). We will elaborate on the significance of these results later in the paper.

Weeks 3-4: Learning to analyze and question. The lack of substantive discussions in both groups made us aware of the need to focus participants' attention to particular aspects of each case. More specifically, we had to emphasize that they investigate other ways of teaching the concept and to analyze the students' work. We used these discussions as occasions to demonstrate multiple ways of sequencing the content using various instructional tools including manipulative materials and graphing utilities. To this end, our goal was to provide them with a model of analytical inquiry that was not available to them.

During the second and third weeks of the implementation phase, we asked students to revisit and re-evaluate the first two cases again. We used the similarity case during week 4 and asked them to compare and contrast the teaching style presented in this case with the previous two episodes. Moreover, we encouraged the participants to question the mathematics that was presented in each case, exploring the same concepts in class.

Weeks 5-10: The build up of a community of learners. We introduced the

Inverse Function case during the fifth week of data collection. During this session the participants immediately began their discussions by reviewing the mathematical argumentation the teacher used. Their subsequent analysis was focused on generating various methods of presenting the concept so to avoid the dilemma presented in the case. A majority of the students admitted that they themselves did not know why the procedure worked and asked if we could explain it to them. Consider for instance the following from the whole group discussion that followed the study of the inverse function case. Notice that during this discussions the participants explicitly question their own claims to "knowing" mathematics or pedagogy.

> Jane: I am looking at this and I am thinking what I would do if this were my class. I mean now that I think about it I have no Idea why we do all this. Does anyone here know?
>
> Silence.
>
> Jim: I guess I never asked why it worked… I found the process easy to remember so I just did it.
>
> Samantha: Okay, I bet this is something real simple, which is why we can't think of it right now. We are thinking too advanced.
>
> Researcher: I am not sure I understand what you mean here. Suppose these were your students, how would you explain to them why they have to follow the algorithm outlined here?
>
> Samantha: Well, they have to know it for later—
>
> Researcher: Like when? Could you give us an example? Can anyone talk about a couple of situations where we actually need to know how to find the inverse functions? How does it help us solve a problem?
>
> Silence.
>
> Researcher: Anyone? Anyone have a different way to explain the mathematical rationale for doing the procedure for finding the inverse functions?
>
> Silence.
>
> Researcher: Is there another way of looking at this without doing symbolic manipulations? Can you think of a graphical representation of inverse functions?
>
> Tony: I only learned it symbolically.
>
> A number of other students confirm Tony's claim.
>
> Tony: I am thinking here that if I don't know it myself how can I teach it to kids—Boy am I in trouble! (everyone laughs)—Seriously, I am wondering now how much of these stuff I know myself, I mean I know that like I can use them and stuff but I am not sure if when they ask me why I would know how to explain.
>
> Sarah: Me too. Not that all of them want to know why—I know from some of my observations that they ask, "why do I need to know this" only to avoid doing math…

Researcher: But is it really what is happening in this case? Are the students asking lazy "why" questions?

During weeks 6-10 of implementing case-study methodology, the participants themselves began asking questions about assessment of learners' thinking and the cognitive obstacles they experience when encountering different topics. During these discussions, they asked us to illustrate different ways of presenting certain concepts to learners.

Final Journal

Once the case studies were completed, during the 13th week of instruction, we asked students to submit a second journal entry reflecting on the same questions we had posed to them at the beginning of the course. Each participant's comments on the final journal was matched against those he/she had made on the first journal entry in order to trace changes in their current thinking and their level of analysis. We then cross-examined both sets of data to determine the common themes that emerged at both sites. We then made implications concerning the use of case studies on these common themes. Table 2 summarizes the results of the final journals.

The results of the final journal entry contrasted with the initial responses we had received from the participants. These were directly related to the goals we had set to accomplish with the use of case studies. The predominant themes included increased attention to children's thinking, an awareness of the need for continued self learning of both mathematics and pedagogy, and the necessity of adjusting and selecting curriculum materials to accommodate specific intellectual needs of the learners. In what follows we will elaborate on these results.

As opposed to their initial reactions to the recommendations of reform, in their final journal 38 of the participants claimed understanding parts of the Standards for teaching and curriculum. Moreover, 44 of them suggested that they were unsure about how to implement the standards in a sustained manner. Forty-three of the participants recognized that the type of teaching recommended by the reform documents was different from their own experiences as learners. They expressed that they needed to learn more in order to implement the Standards successfully. Only four of the participants at both sites felt confident about enacting Standards-based teaching.

A majority of the participants (n=46) expressed concern about their ability to balance curriculum coverage and nurturing students' conceptual understanding. For this group the challenge of organizing classroom interactions to foster advanced mathematical thinking through discourse was more obvious than in the beginning of the semester. Nonetheless, 46 of the participants found reform-based teaching valuable.

All participants expressed concern about their ability to understand, analyze, and respond adequately to students' thinking and questions. Forty-seven of the participants were apprehensive about their own ability to make sense of student-

Table 2
Summary of Results from the Last Journal Entry

Themes	Site 1 (n=27	Site 2 (n=23)	Total (n=50)
Analysis of Standards			
I understand some parts of it	20	18	38
Not sure how to implement it long term	21	22	44
Teaching the way standards promotes is different from what I am used to	24	20	43
I will need a lot of help and resources to implement this type of teaching	20	22	42
I am concerned about how to sequence my instruction so to cover my curriculum and nurture students' thinking	24	22	46
I am still not sure how to facilitate discourse	10	14	24
I want to be that kind of teacher	26	20	46
I am confident I can do it	3	1	4
Analysis of self strengths and weaknesses			
I am now familiar with a lot of resources	25	20	47
I am afraid of falling back to my old routine	20	16	36
Being pressured by curriculum coverage	22	19	41
Management and motivating kids	15	12	27
What if I don't get what they say in class	25	22	47
What if I don't know the answer to their questions	26	23	49
I thought I knew it all but now I know that I don't know "why" certain things work	23	21	44
Concerns for teaching and challenges they anticipated			
Making sense of kids' talk	25	19	44
Motivating all students	24	20	44
Having to work with traditional textbooks	17	19	36
Standardized testing/tensions with new teaching methods they wanted to implement	7	2	9
Attributes of a good teacher			
Able to listen to students	26	23	49
Caring	27	23	50
Knowing mathematics: why and how questions	26	23	49
Flexibility	25	20	45

generated arguments. Moreover, 44 of the participants were concerned about whether they could justify and explain mathematical procedures. Forty-nine of them sensed a conflict between their need for being the authority in class (knowing the answers) and acting as a facilitator of learning. We found this point of particular significance since it contradicted their response at the beginning of the course. Initially, all participants had assumed great confidence in their knowledge of mathematics and their own ability to teach. They were now questioning what they knew and how they would present the subject matter to their students.

In contrast to their initial judgment on the attributes of a good mathematics teacher, at the end of the study a majority of the students identified the ability to listen to the students as a significant quality for a good mathematics teacher (n=49). Although being “caring” remained a primary attribute for all of the participants (n=50), the teacher’s own mathematical ability, knowing why, and how certain algorithms work became an equally important consideration for them (n=49). In addition, 45 of the participants identified the teacher’s flexibility in working with children’s strategies and mental structures as an attribute of a good mathematics teacher.

Conclusions and Implications

The purpose of the research reported here was to study the impact of using case analysis method on professional growth and development of 50 prospective secondary mathematics teachers. Our results support the use of case studies in teacher preparation (Shulman 1986, 1992). Indeed, in light of our findings we suggest that teacher education programs need to make more efforts to encourage, stimulate, and disseminate case materials with the intent to advance inquiry into learning about teaching. The findings of the study indicate that the use of case-analysis methodology was effective in increasing the participants’ knowledge about problems of practice, in raising their sensitivity towards student learning, and in motivating them to think in greater depth about efficient teaching strategies. It assisted them in gaining a broader understanding of the recommendations of reform and the complexities associated with teaching in ways that build around student inquiry. Initially, all participants had assumed reform-based instructional practice simple to implement. Later they came to critically examine characteristics of innovative teaching.

The use of the case-analysis method also provided opportunities for the participants to revisit mathematics and engaged them in mathematical problem solving as learners. This particular result stands in complete contrast with the findings from previous studies on the impact of the use of case studies on teacher development (Lampert & Ball 1998). Lampert and Ball contended in the course of their pedagogical investigations of case studies, that the teachers with whom they worked rarely addressed mathematics. In addition, many of their students’ normative assumptions about teaching were not challenged in the course of their examination of the case studies.

The use of case studies helped the creation of a learning community in which the participants struggled to make sense of mathematics and the pedagogical problems they explored. In the course of their discussions and analyses they explicitly contributed to each other’s development by challenging each other’s interpretations. The particular strengths and weaknesses of the individuals in each of the groups both enhanced and impeded the discourse of the group relative to the cases they analyzed. In places where the members of the group shared the same mathematical deficiency, their analysis of the case remained artificial. They neither individually nor collectively attempted to understand the mathematical perspective presented in the case.

Although it is not by any means our intention to make claims to having changed the participants' beliefs about teaching, learning, and the nature of mathematical knowledge, as that was not the focus of the work reported here, data indicates that the experience of examining case studies was useful in providing the participants with a lens through which they could view classroom events and interpret them. Allowing the future teachers the opportunity to experience and gain facility in using this lens should be a priority in mathematics teacher education.

As discussed earlier in this paper the use of case study method did not immediately result in substantive analysis of either instruction or curriculum on the part of the participants. In fact, it became clear to us that in spite of their educational background, the type of reflective analysis we had expected was a novelty to many of them. In this study there was a serious need to model for the participants how to critically analyze a pedagogical problem. We had to demonstrate in class, as we collaboratively reviewed written and video case studies, what questions about student learning would be useful to ask, what signs to look for when evaluating classroom interactions, and how to assess the validity of pedagogical conjectures they made. This finding is of vital importance in mathematics teacher education as it speaks to the fact that change in teachers' thinking does not occur simply by placing new materials in their hands or placing them in school settings but by guiding their intellectual growth in a sustained manner.

Although data indicates that we succeeded in creating dissonance in participants' thinking about teaching and what they viewed as valuable knowledge for successful teaching, we are concerned about the short- and long-term impact of our efforts on their practice. For the participants in this study this development occurred during the final phase of their teacher education program. As the data indicated, we managed to focus their attention on cognitive aspects of teaching and made them aware of the need for continued learning. However, our time with them was limited. We believe these issues should be addressed earlier and continually throughout their teacher education program.

Lastly, it is critical to frame the case-study methodology within a more global perspective in teacher preparation. In this work, the methods course activities were designed so complement the case-study methodology. These activities exposed the participants to the type of experiences that could help them better understand the content of the NCTM's standards and their implications for teaching and learning mathematics. These included: conducting mathematical explorations using technology and various types of manipulative materials, analyzing and evaluating reform-based middle and high school textbooks, conducting clinical interviews with school children and their peers, writing lesson plans and units of instruction, and analyzing case studies. These experiences, coupled with the case- analysis method proved successful in assisting the participants develop deeper knowledge on professional issues. In the absence of these combined experiences we are unsure whether similar positive results would have occurred.

Notes

[1] Later in the semester the students were asked to select a particular topic from the collection of cases we presented them and to build a unit of instruction that addressed the teaching and learning dilemma presented in it.

[2] This is a very legitimate concern as the Principles and Standards for School Mathematics propose that certain mathematical strands to be covered in elementary, middle, and high school levels. The specifics of what and where these strands should be addressed are not specified. We were surprised that only 5 students raised concerns about this issue.

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Appendix

Case I: Fractions

As Terry completed her discussion of the procedure for dividing fractions, Johnny raises his hand and suggests that he does not really understand why he has to invert the second fraction and multiply by the first one. He suggested that he has a simpler way of dividing fractions and wants to know whether he is right.

Johnny: Here—this is what I am doing—If I have $\frac{3}{5} \div \frac{7}{10}$

I will do this: $\frac{3}{5} \div \frac{7}{10} = \frac{6}{10} \div \frac{7}{10} = \frac{6 \div 7}{10 \div 10} = \frac{6 \div 7}{1} = \frac{6}{7}$

I can do this for any two fractions:

$$\frac{3}{6} \div \frac{7}{9} = \frac{27}{54} \div \frac{42}{54} = \frac{27 \div 42}{54 \div 54} = \frac{27 \div 42}{1} = \frac{27}{42} = \frac{9}{14}$$

How are the two algorithms similar?

Sunny, another student argues that she does not understand why Johnny's method does not work for multiplication of fractions. She, too, asks Terry to tell her why she couldn't use a similar procedure for finding the product of fractions.

Terry has never seen this algorithm before and does not know if it is always true. She does not know how to respond to Johnny and Sunny.

Case II: Ratios

Donna provides students with a handout. On it, they are given pairs of ratios and they must decide on whether the ratios are equal. If they identify the ratios as unequal, then they must identify which ratio is bigger than the other. Following a 15-minutes individual sit work, she asks students to come to the board, and to present their solutions to each of the assigned exercises. Following each presentation, Donna opens the discussion to the entire class and asks if they agree or disagree with the answer that the presenter offers the group.

The first two pairs of ratios on the handout are equal. Donna asks students if they agree or disagree. Both presenters had used factoring to simplify the larger ratio to create an equivalent ratio.

Donna volunteers Jenny to go to the board and to solve the following exercise:

3/4   5/6

Jenny: I said 5/6 is bigger than 3/5.

Donna: Great, Jenny. What do you think? (Pause) do you all agree with Donna? Do you think 5/6 is bigger than 3/4?

Artoro: I said, I said—(pause) I think she is wrong cause we have—I think they are equal-

Donna: How do you know that Artoro?

Artoro: I drew the picture here, see with 3/4 we have one piece left, with 5/6 we have one piece left . . . so for . . . I mean in both pictures each of them has a piece left, so they have to be equal.

Donna steps back and looks at the picture Artoro showed her. She draws the diagrams on the board, and rather than telling him he is wrong, she asks the group to decide on which of the answers is wrong.

Jenny: But see that is not right— ow can it be right—see 5 is bigger than 3, and 6 is bigger than 4. So, both numbers are bigger so this one is bigger (pointing at 5/6).

Donna: I think we have an interesting case here. We have two answers here and now we have to decide which one is right—well . . . I mean, can both of these answers be right?

Eddie: No, how can both answers be right? We can vote, Ms!

Donna: But, I want to know why you think either one is right . . . I want to know what you think . . .

Silence

Donna: It seems to me that Jenny and Artoro have different arguments for why the two could be equal or not equal… which one of them do you believe, or if you disagree with both of them, tell us what you think. Tell us your answer.

Terrance: But see in that picture (point at the 1/6) we have that little piece left over—and that in that other picture (point at 1/4) we have this piece but when we look at them, that one—I mean 1/4 is bigger than the, the, (Donna says 1/6). Yeah, 1/6.

Donna: So, what are you saying? Are you agreeing or disagreeing with them?

Terrance: I guess I am saying I disagree – I think, I think 3/4 is bigger than 5/6 cause the leftover piece is bigger.

Case III: Similarity

Prior to this session students had spent nearly a week on several investigations that introduced them to the formal definition of congruent figures. The concept of similarity of shapes had not been introduced explicitly in the textbook, though children were assigned to review a set of figures and asked to determine which they thought were similar. Following this activity, students were asked to articulate what they thought it meant to say that two figures were similar. As students were assigned to work on this question, Gina walked around the different groups and reviewed their responses to the question. A number of students had suggested that squares and rectangles were similar, and that if two pictures looked alike they could be considered as similar. Naturally, students' responses did not include detailed attention to accurate measurements and to proportionality of the sizes of the similar shapes. Considering students' raw generalizations Gina decided to stop the small group activities and began a whole group discussion on the concept of similarity. The following is a verbatim transcription of the conversations that took place in her classroom during that discussion.

Gina: What is the best way to create two congruent figures?

Many students raise their hands. Gina asks one student to respond.

T: I can trace the figure.

J: Yeah, like put a piece of paper on top and just copy it... we know that they are going to be the same thing.

Gina: Let's do that together and see what we get— She places a transparency slide on the overhead projector and draws a rectangle. Placing another transparency on top of it she traces the rectangle with a different color marker.

Gina: Okay— Let's see— what do you notice about the two shapes that we have here? (pause) How are they similar?

K: They look exactly the same—

Gina: What else? Tony?

Tony: I don't know— they are just the same— Like they are just copies of each other.

Gina: Let's go beyond that and try to describe what we see in terms of what we have talked about in the past— Like, what can we say about their angles? What can we say about their perimeters? What can we say about their areas?

Students shout out that they are the same.

Gina: Right you guys— This is really cool— Their areas are the same, their perimeters are the same, their angles are the same, and the length of their sides are the same— have I missed anything Jacquie?

J: Nope—

Gina: Good— Now I want you to think about situations when we have two things that look alike but are not exactly the same size— Let's forget the examples in the book and think of real life situations— What do you think? (pause)

S: Like in the movies?

L: Oh, yeah— in the pictures too.

Other students nod their heads in agreements.

Gina: Cool! So, when we go to the movies what we see on the screen is actually similar to those images that we have on those little films... Why do you say they are similar?

H: They are the same people only bigger—

Gina: tell us more Hillary—

H: Well, it is like Tom Cruise but a whole lot bigger— His face is bigger, but the same thing— His body is the same thing, only bigger—

Gina: How about details in his clothing, his face?

T: Well, it has to be the same only bigger—

Gina: Right— All the detail is there only bigger— But could we enlarge his legs say 10 times and his head, I don't know, say only 5 times?

Silence—

Gina: What would be wrong with that?

Jenny: It won't be right— I mean, he won't look right— Not as good looking as Tom Cruise (laughs)

Gina: What else? Who else has something to say about this?

K: I think that it won't look right because if we make his legs bigger say ten times then we need to make his head also ten times— I guess that is what it is— If we don't enlarge it the same way for all parts it is not gonna look right— It won't look like him anymore—

Gina: What do you think Sam?

S: He would sure look funny- (laughs) I think some parts of him will be similar but not the whole thing—

Gina: How can we make sure that when we have two shapes, they are similar?

P: I guess we can check to see if it is bigger or smaller the same amount on all parts.

Gina: Good idea— Let's see if we can do what P. said—

She turns on the overhead projector and projects the rectangle she had drawn on the transparency on the screen—

Gina: Do You think these two shapes are similar (pointing at the one of the screen and the rectangle on the transparency?

D: I think so—

Gina: How can we be sure?

Case IV: Inverse Functions

Sarah's objective for the instructional period was to introduce the topic of inverse function. She began her lesson by asking students if they remembered how to "find the composition of two functions," as she had discussed the topic only the day before. One student volunteered,

Sue: It is when you write the two functions and then you think of the second one as like the value of the first one— Umm, can I give you an example?

Sarah: You are on the right track Sue, why don't you tell me what you are thinking and I will put it on the board—

Sue: Okay, let's say we have f(x)=x2 and then g(x)=2x+1, now say we want to find fog then we say f(g(x)) equals and then for x in x2 we put (2x+1)—

Sarah: Very good— Aha!

Sue: So we get (2x+1)2.

Sarah: What do we do to this now?

Sue: We foil it?

Sarah: Excellent— did you all see what she did— That's how we find the composition of two functions— Now, if we were asked to find (gof(x)) what would we do? (pause) Jack?

Jack: umm

Sarah: Do you need to think about it?

Jack: Aha!

Sarah: Okay— (pause)— Randy?

Randy: I think we plug x2 for x in g(x), right?

Sarah: That's right— [she turns around and erases the board]— Let's do one more of these together just to make sure we know what we're doing (she writes f(x) = 3x2-4, g(x)=1/x on the board)— Now I want everyone to find both the fog(x) and gof(x)— You have about 5 minutes to do this. If you have questions, raise your hands—

Tanya: I have a question about homework.

Sarah: I will get to that later.

As several students raised their hands for assistance, Sarah decided to save time and to do the problem again on the board so that everyone could see the solution method. She solved both questions for students. At this point Sarah looked at her watch and noticed that she has lost 20 minutes of her instructional time. She announced that she needed to proceed with a new topic.

Sarah: Now, today I want to talk to you about inverse functions... (Erasing the board as she talks) Who knows what I mean by inverse function?

Sue: Is it like one over a function?

Sarah: (laughs) I knew you were going to make that mistake— A lot of people think that when you say inverse function it is like fractions and when we found the reciprocals of them... NO, this is a little different... (pause).. I am going to give you a definition and I want everyone to write this in their notebooks... (she turns around to start writing on the board)— If you get a little confused by the definition don't worry I will explain it to you after I am finished writing. [She writes the definition directly from the book and reads it aloud as she is writing on the board)—

Sam: I don't get it!

Sarah: I told you it might be hard for you to understand, let me finish it then I explain (she finishes writing).

Okay, now— It is like this— Say I give you a function like: 5x+1 and then I ask you to find that inverse of this function. The first thing you need to do is to write this as y=5x+1, then like we did for linear equations solve it for x, in this case, we fist subtract the 1 then we divide by 5, and what do we get? (y-1)/5, this is the inverse of our function— To see if we are right, all we have to do now is to compose this function on the origin one— If we get x, then it means that we are right and that this one is the inverse function— So, let's do it.

Peter: I don't get it— Why do we have to get x?

Sarah: It is okay, I will review what I did in a minute and then do a couple of more examples.

[Peter is completely disengaged and starts throwing his pencil at the person sitting next to him Sarah notices this and asks Peter to pay attention.]