# Questions To Ask and Issues To Consider While Supervising Elementary Mathematics Student Teachers

**By Randolph A. Philipp**

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Recently I was asked to make a presentation to supervisors of elementary school student teachers about the teaching of mathematics. I structured the presentation around a set of questions the supervisors might ask student teachers about the mathematics lessons they observe. Before sharing these questions, however, I must address whether there is a need for such questions. After all, have the students not completed an elementary mathematics methodology (EMM) course in which they learned how to teach mathematics? I begin this paper with a vignette, drawn from my EMM course, that illustrates that beginning student teachers have a wide variety of beliefs, values, and knowledge about the teaching and learning of mathematics. If supervisors continue to challenge the student teachers' assumptions and practices about the teaching and learning of mathematics, those student teachers may develop more effective ways to teach mathematics.

## Vignette I

### *Learning about My Preservice Elementary School Teachers*

An event that occurred half way through one elementary mathematics methods course caused me to think very differently about what I was accomplishing. The day's lesson began with a discussion about introducing multiplication using base-ten blocks. After the discussion I asked a student who had been considering how to teach multidigit multiplication if she wanted to share her thinking with the class. Karen (all names used are pseudonyms) was glad to try out her ideas. Rather than teach the traditional algorithm, Karen wanted to work with an expanded algorithm because she thought it would engage her fourth-grade students in making sense of the algorithm and because it focused on place value. The traditional algorithm and an expanded algorithm (both displayed in Figure 1) were placed side by side on the chalkboard.

**Figure 1**
Two Ways To Multiply 23 x 22

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   23            23
 x 22         x  22
 ----         -----
   46             6
  460            40
 ----            60
  506           400
              -----
                506
```

Two students disagreed about which approach is better: Betty stated that the standard algorithm is the best approach whereas Sarah reasoned that students might learn to apply the algorithm but would not understand it. A transcript excerpt follows:

> *Betty*: It makes sense to do the standard algorithm. Just tell them that it is 460 because it is 20 x 23 [when multiplying by the 2 in the tens place].
>
> *Randy*: So you would just tell them?
>
> *Betty*: Yeah.
>
> *Sarah*: To teach with understanding, you must start with the long way. [She compares doing long division before short division.]

*Randy*: Betty said, "This [standard algorithm] makes the most sense. Just help them understand that this 2 is 20, and they'll understand." What about that?

*Sarah*: It doesn't make sense if you teach it [the traditional algorithm] to them first.

*Randy*: Why?

*Sarah*: Because they won't understand place value; they're just doing the 3 times 2, then 2 times 2. . . .

*Betty*: I would *tell* them. All you have to do is tell them.

*Sarah*: Even if you tell them, it is not going to be a visual enough thing, and if you tell them, they are going to go, "Uh huh, uh huh (nodding her head)," and they are still going to be doing [the algorithm without understanding place value].

*Randy (summarizing)*: Betty says that the standard algorithm makes sense; you can just tell them the 2 is really 20. Sarah says that they will listen but not understand, and it is best to do it the long way first.

At this point many students have raised their hands to respond to the discussion. Following are some of the responses:

*Kevin*: I don't see what the problem is with working backward. You can show them how to do it [the standard algorithm] so that they get the right answer, and then you can always go back and dissect it. I don't see why you can't show them the process. Show them the process, the easy process, and let them have some success with it, and then you can go back and dissect it.... As long as the process gets done and as long as the meaning gets to them later on, I don't think it really matters.

*David*: Same as him [referring to Kevin]. Why not work backwards? Why not start with the easier one [the standard algorithm], and at first maybe they won't have the concept of "Why do I put the zero here?" Just tell them to "*Put it here*." Get them rolling with it.

David goes on to explain that even if students do *not* know place value, they can still have success with the traditional algorithm, but not with the expanded algorithm.

*Francis*: I agree with what David said, because hopefully by the time all of your children exit fifth grade they will know place value, but if not—if some of them don't—they still will know double-digit multiplication, which they are going to use throughout life. I didn't know that concept when I was in fifth grade. I did fine throughout math. I knew how to do double-digit multiplication. I think that *that* is the most important thing, that they know how to *do* it.

*Carol*: I just remember going through school and I *never* knew, like 20 was in the tens place, in *that* form. It was college that I figured that out. It was like a clue-in. 'Cause I was doing algorithms, and that's how I got the answer; I was doing everything separately. So, I don't know if it *is* that important for the child to know exactly. 'Cause I got through, all through my years not knowing that, and in college I went, "Oh!"

I understand how difficult it is for teachers to teach conceptually, and I was prepared for many of my students to tell me that if given the opportunity they would probably teach the traditional algorithm to their students. What I was not prepared for were the reasons they gave. I was prepared for the students to say that they would like to teach conceptually but that they felt constrained, either by their external circumstances or by their lack of confidence. But instead, many said that success at the algorithm was of primary importance, and after their students learned the algorithm, then they might come back and help them learn the concepts. Because of the goals I had held for the lessons to this point in the course and the activities that for me promoted those goals, I was surprised to hear many of my students respond this way.

I share this story, first, to acknowledge that the process of learning about what one is and is not accomplishing in teaching is an issue of constant concern. Second, these students were *my* elementary mathematics methods students, and the beliefs many of these students held would not support the kind of mathematics learning I value. How is one to supervise such students? Instead of viewing these students as incapable of teaching mathematics meaningfully, supervisors who view them as being in the process of growing and evolving will be able to more effectively interact with and support these students.

## Four Questions

I hope the following four questions might help supervisors who want to support their student teachers' development as mathematics teachers. These questions mean little without providing rationale and examples designed to illustrate what I have in mind with each question, and the examples I provide were drawn from the research literature and from my own experience. Just as these questions are of little value without the rationales and examples accompanying them, student teachers' responses will be useful only if accompanied by specifics that arise from careful consideration of the complexities of their teaching and their beliefs about mathematics teaching and learning. The first two questions are about content, the third is about methodology, and the last is about assessment.

## Content

(1) What mathematical *concepts* are you teaching? What mathematical *procedures* or *algorithms* are you teaching?

Data from the Third International Mathematics and Science Study (TIMSS) indicate that in the United States today most students are being shown how to *do* mathematics but not how to *reason* about mathematics (Stigler & Hiebert, 1997). Mathematics educators are interested in both, and although I think there is a place in mathematics for learning procedures and algorithms, I think that these areas have been overemphasized at the expense of understanding of the underlying concepts.

In the literature are numerous examples of students learning mathematical procedures without also learning the underlying concepts, and after sharing some examples from my own work in schools, I will turn to some of that literature. Vignette 2 portrays one conversation I had with a child in first grade and another with a child in fifth grade. The two children are successful in their respective classes. Notice how the first-grader's reasoning is built around an image she has developed from one experience with pattern blocks, and notice how the fifth grader seems to have no image on which to draw and seems instead to be trying to recall the appropriate way to manipulate the symbols.

## Vignette 2

### *The Consequences of Learning Procedures Without Understanding: An Example from Fractions*

For the past few years I have taught weekly in classrooms at a local public school. One October I introduced fractions to first-grade students by having them work with pattern blocks as they constructed wholes using red trapezoids (halves), blue rhombi (thirds), and green triangles (sixths). By the end of the lesson, some students were constructing wholes using different-sized pieces. At this time we did not represent any of the fractions symbolically; instead we just spoke about the fractions.

The very next day I was at my son's soccer practice when two mothers asked me what I was doing in the first-grade class. Alyson, a student from the class, happened to be standing close by, so I called her over. Alyson and I talked:

*Randy*: Alyson, is a half and a half more than a whole, less than a whole, or equal to a whole?

*Alyson*: What does *equal* mean?

*Randy*: *Equal* means "is the same as."

*Alyson*: It is equal to a whole.

*Randy*: Is a third and a third more than a whole, less than a whole, or equal to a whole?

*Alyson*: Less than a whole.

*Randy*: How do you know?

*Alyson*: Because you need another third to make it a whole.

*Randy*: Is a half and a third more than a whole, less than a whole, or equal to a whole?

*Alyson (after about five seconds)*: Less than a whole.

*Randy*: How do you know?

*Alyson*: Because you need another sixth to make it a whole.

*Randy*: Alyson, how did you think about that question? Are you picturing something in your mind?

*Alyson*: A red and a green.

*Randy*: You mean you know a red is a half and a green is a sixth?

*Alyson*: Yes.

Notice how quickly Alyson was able to make the link between the physical model and the fractional names. Our schools are filled with first graders who, if given the chance, are ready to reason as Alyson did.

Later that year I was escorted to a class by a fifth grader who told me that his class had been studying fractions, so I posed the same questions I had asked the first grader. Our conversation went as follows:

*Randy*: Jim, is a half and a half more than a whole, less than a whole, or equal to a whole?

*Jim*: Equal to a whole.

*Randy*: Is a third and a third more than a whole, less than a whole...?

*Jim (Interrupting)*: It is two sixths.

*Randy*: Hm. That's interesting. When I asked you what one half and one half was, you said one whole, but when I asked about one third and one third, you said two sixths.

*Jim*: Hm. Oh, ok, I see now.

*Randy*: What is it that you see?

*Jim*: One half and one half is two fourths.

Unlike the first grader who had a way of picturing halves and thirds in her mind, Jim primarily thought about these fractions by considering the symbols and procedures. As a result, he would manipulate the symbols in his head, but he had little experience developing number sense to support his thinking.

Alyson was mentally manipulating pattern blocks whereas Jim was mentally manipulating symbols. Alyson's solution was correct whereas Jim's was not. What differences might account for Alyson's success and Jim's difficulties? One difference that may appear significant is that Alyson used manipulatives to associate meaning with the fraction names. The use of manipulatives is, however, not a magical cure-all; Deborah L. Ball (1992) has described situations in which students used manipulatives without learning the mathematics intended by the teacher. A more important difference involves the norms established in each classroom. Erna Yackel and Paul Cobb (1996) distinguished social norms, which are general and might be established in any classroom, from sociomathematical norms, which are norms particular to a mathematics class:

> To further clarify the subtle distinction between social norms and sociomathematical norms we offer the following examples. The understanding that students are expected to explain their solutions and their ways of thinking is a social norm, whereas the understanding of what counts as an acceptable mathematical explanation is a sociomathematical norm. Likewise, the understanding that when discussing a problem students should offer solutions different from those already contributed is a social norm, whereas the understanding of what constitutes mathematical difference is a sociomathematical norm. (p. 461)

Jim behaved as if for him the norm when working with fractions was acceptance of algorithms over reasoning. Given that Alyson had not yet learned any algorithms for fractions, such a norm could not yet have been established for her.

As a basis for considering conceptual and procedural understanding, I present one view of what is meant by *to understand*. James Hiebert and Thomas P. Carpenter (1992) summarized the research in cognitive science and in mathematics education to present a model of *understanding* as a network of relationships between and among concepts, procedures, and facts. Mathematical concepts, procedures, and facts are understood by an individual insofar as they are integrated into that individual's network of mental representations, and the degree of understanding is determined by the number and strengths of the connections among these concepts, procedures, and facts. In other words, one way to learn is to develop new connections among already existing mental nodes. Learning mathematics then should involve the development of such links. Vignette 3 illustrates the incomplete development of links between a second grader's understanding of place value and his knowledge of the standard algorithm.

## Vignette 3

### *A Second Grader Subtracting with (and without) Place Value*

One morning over breakfast my older son, Elliot, a second grader, was trying to determine "how many games over 500" each major league baseball team was, by finding the difference between the numbers of games a team had won and lost. At the time the Atlanta Braves had the best record in the National League with 38 wins and 14 losses, and Elliot was having trouble figuring out how many more games they had won than lost. I reminded him that he could think of this as subtraction, and he correctly solved the problem using the predominant American algorithm for subtraction. Elliot's younger brother, Andrew, who had just turned five, also wanted to solve some subtraction problems; I gave him problems he could solve (e.g., 9 - 4 and 10 - 6). When I left the room and Andrew ran out of problems, Elliot decided to play the role of teacher; he gave Andrew the subtraction problem 4 - 5. As I reentered the room, Elliot was explaining his solution to his younger brother. His work is shown below:

$$\begin{array}{r} {}^{1}\,3 \\ \not{4} \\ -\ 5 \\ \hline \end{array}$$

Elliot had learned this procedure in second grade, but he was confused as to when to apply it. Although it might seem that Elliot had learned subtraction without developing the important supporting place value knowledge, that was not the case. Later that same day Elliot, riding his bicycle, accompanied me when I ran. I posed math questions to him: 100 - 60, 100 - 30; then 100 - 63, 100 - 46; then 1000 - 600, 1000 - 300; and finally 1000 - 420. He correctly figured the answer to each question using only "his head."

Elliot had learned the subtraction algorithm and could apply it when his teacher posed questions. He had also developed enough place value knowledge to subtract 63 from 100 and 420 from 1000 without applying a written algorithm. Yet his understanding of these ideas was fragile, and when he wrote a problem for his brother, he constructed one that looked similar to problems he had seen but was different in a way that was evidently not significant to Elliot. What happened in this case? First, note that Elliot was in a novel situation; in school one seldom asks children to construct problems and even less often asks students to construct problems to be used with younger children. Second, Elliot knew that his younger brother was comfortable with small numbers in subtraction problems, and perhaps that was on his mind when he wrote 4 - 5. In any case, when faced with the need to subtract 5 from 4, without drawing any connection between his procedure and his knowledge of place value, he invoked his "cross out the number, reduce by 1, and put the 1 there" procedure, which had worked for him in the past. Because there was no number "next door" to cross out, however, he crossed out the *only* digit in the minuend. Elliot has in place many of the pieces related to multidigit subtraction, but the connections among these pieces are not yet strong.

Having rich and strong mental connections among concepts and procedures is powerful because it is generative, enhances transfer, promotes remembering, and reduces the amount to be remembered (Hiebert & Carpenter, 1992). These connections develop gradually and differently in each student. If educators believe in the importance of students' developing connections among the conceptual ideas and the associated procedures, then they are left with a question: What is the relationship between developing conceptual knowledge and procedural knowledge? The relationship is complex and not completely understood. Researchers who study children's thinking have found that students can learn to perform procedures and apply algorithms without understanding the underlying concepts. Consider a result from the Second National Assessment of Educational Progress (NAEP) in Mathematics:

> In general both of the younger age groups performed at an acceptable level on knowledge and skill exercises.... Students appear to be learning many mathematical skills at a rote manipulation level and do not understand the concepts underlying the computation. (Carpenter, Kepner, Corbitt, Lindquist, & Reys, 1980, p. 47)

For example, whereas 70 percent of 13-year-olds correctly performed a certain division computation in the second NAEP, only 40 percent correctly solved a word problem that required dividing the same numbers used in the computation. Students' greater success with knowledge and skill exercises than with conceptual understanding is a continuing trend, evidenced by subsequent NAEP reports (Lindquist, 1989). This finding is not surprising inasmuch as whether teachers are surveyed (Mullis, Dossey, Owen, & Phillips, 1991) or observed (Eisenhart et al., 1993; Stigler & Hiebert, 1997), they are found to focus on teaching facts and procedures much more than on teaching reasoning.

An example illustrating the complex relationship between conceptual knowledge and procedural knowledge, drawn from the Cognitively Guided Instruction literature (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Fennema, Franke, Carpenter, & Carey, 1993), involves a second grader who was presented 70 - 23 written vertically. Following a common misapplication of the traditional subtraction algorithm, she responded, "Fifty-three." She was then asked whether she could work the problem another way, and using both base-ten blocks and then a hundreds chart, she arrived at 47. She was clearly confused by these conflicting answers, and when she was asked by the interviewer which she thought was correct, she pointed to the 53, the result from applying her procedure. After rechecking her calculation, she said, "This [algorithmic solution] *has* to be right." This child's second-grade teacher had not taught the algorithm but had instead encouraged the students to use strategies with which they were comfortable, strategies that made sense to them. The child's father, a student teacher at the time, thought that his daughter should have learned the algorithm, so he taught it to her. Fortunately she had previously developed the place value concepts necessary to solve such problems (several days later the child told the interviewer that she understood her error), but this episode highlights the importance that students ascribe to the use of algorithms in mathematics.

Another consideration related to teaching mathematics concerns the beliefs that students develop. A teacher enrolled in a mathematics education master's program observed that her students asked *why* when learning science and social studies, but they never asked *why* when learning mathematics. She asked her best mathematics student about this observation, and he explained, "Of course you don't ask *why*. In mathematics you only have to know *how*." In contrast, consider a study of first graders drawn from six classrooms in two different school systems (Franke & Carey, 1997). All the teachers in these six classrooms provided their students with opportunities to solve a variety of addition, subtraction, multiplication, and division problems presented in context. The teachers expected students to solve problems in various ways, and they expected the students to explain their reasoning. Thirty-six children from these classes were individually interviewed to determine their views about mathematics and the teaching and learning of mathematics, and responses were coded into categories. When students were asked "What would you tell kindergartners about what math is like in first grade?" 28 (78 percent) students' responses were coded into the "solving problems" category, 14 (37 percent) students' responses were coded in the "talking about mathematics" category, only two (6 percent) students' responses were coded in the "focusing on correct answer/paper and pencil tasks" category, and one (3 percent) student's response was coded in the "speed and accuracy" category. When asked what their teachers would want to know about solving a problem, 36 (100 percent) students responded that their teachers would want to know how they solved the problem and none responded that their teachers would want to know their answers. When asked who should decide if a student's answer is correct, three (8 percent) said that the teacher should, 28 (78 percent) said that the students themselves should, and five (14 percent) said that both the teacher and students should decide. On the basis of these data, Megan L. Franke and Deborah A. Carey concluded that if children are in classrooms in which (a) problem solving is valued, (b) instruction is in the spirit of the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards* (1989), and (c) teachers have and use knowledge of children's mathematical thinking, then they "hold different perceptions about what it means to do mathematics from those traditionally held by students" (p. 23). This study shows the effect of the instructional environment on students' beliefs about mathematics.

The first question highlights the principle that when teachers and students adopt the stance that what is taught in mathematics can be understood meaningfully and not just memorized as a procedure, the elementary school mathematics content is seen in a different light. When teachers teach their students algorithms without teaching the mathematical concepts embedded in the algorithms, children will learn to *do* mathematics without *understanding* it. However, when teachers view mathematics as an integration of concepts and procedures and create a mathematical learning environment in which their students are expected to make sense of mathematics, they will help their students develop mathematical understanding that can transfer to other contexts within and outside of school.

## Content

> (2) Are the concepts and procedures part of a unit? If so, then in what order should concepts and procedures be taught? How do the ideas presented in today's lesson build upon what you did yesterday, and where will this go tomorrow?

Learning takes time, and most of the important things one learns are learned not in a day but over longer periods of time. So is it with mathematics, especially if educators come to value the kind of understanding described heretofore. A student may be able to learn the multiplication algorithm in one day, but the place value knowledge required for students to understand the multiplication algorithm must develop over time. Can teachers, when thinking about the mathematics they want to teach, consider how the ideas and procedures might unfold over the course of the unit?

A critical question to consider when thinking about the development of ideas in a unit is whether the order of development of conceptual knowledge and procedural knowledge matters. The verdict is still out on this question, although evidence indicates that the order is important. As part of a study designed to test a theory for how students develop competence with written symbols of decimals, Diana Wearne and Hiebert (1988) provided students with a treatment comprising nine lessons designed to help them develop conceptual understanding of decimal fractions. In analyzing the task 2.3 + .62, Wearne and Hiebert compared performance of the 14 fourth and fifth graders who had no previous instruction in decimals to the performance of the 15 fifth and sixth graders who had previous instruction in decimals. Of the 14 children with *no* previous instruction in decimals, none completed this task correctly before the treatment and 11 (79 percent) completed it correctly after the treatment. Of the 15 students *with* previous instruction in decimals, one completed it correctly before the treatment and six (40 percent) completed it correctly after the treatment. Wearne and Hiebert concluded, "Prior instruction that encouraged the routinization of syntactic rules seemed to interfere with, and prevented the adoption of, semantic analyses of the affected tasks. Apparently, it is difficult for semantic processes to penetrate routinized procedures" (1988, p. 380).

Nancy K. Mack's (1990) findings were similar when she conducted individual teaching experiments designed to help eight sixth-grade students build upon their informal knowledge of fractions. Mack defined *informal knowledge* as "applied, real-life circumstantial knowledge constructed by the individual student that may be either correct or incorrect and can be drawn upon by the student in response to problems posed in the context of real-life situations familiar to him or her" (1990, p. 16). Mack found that students' knowledge of procedures often prevented them from drawing on their informal knowledge of fractions, and she found that, at least initially, students often trusted answers obtained by applying faulty procedures more than those obtained by drawing on informal knowledge. For example, when asked how to add fractions, Tony said, "Across. Add the top numbers across and the bottom numbers across." Mack then asked Tony how much pizza he would have if he had 3/8 of a pizza and she gave him 2/8 more of a pizza. Tony responded:

> Five-eighths. (Goes to his paper on his own initiative and writes 3/8 + 2/8 =, gasps, stops, then writes 5/8.) I don't think that's right. I don't know. I think this (the 8 in 5/8) just might be 16. I think this'd be 5/16. (p. 27)

All eight students in Mack's study attempted to resolve inconsistencies by applying knowledge of faulty procedures; with careful assistance from Mack, students could overcome the interference of faulty procedures, but not easily.

Anton S. Klein, Meindert Beishuizen, and Adri Treffers (1998) have presented additional evidence to support the notion that the order in which procedural knowledge and conceptual knowledge develop matters. They compared classes of second-grade students in two mathematics programs: In the Gradual Program Design (GPD) classes, procedural computation was initially emphasized in instruction; the Realistic Design Program (RPD), however, was designed to stimulate "flexible use of solution procedures from the beginning by using *realistic* context problems" (p. 443). At the end of second grade, the RPD students, who were given freedom to choose their own solution strategies during instruction, were more efficient in choosing solution strategies and just as competent in accurate computation compared with the GPD students, who had been drilled on specific strategies.

If educators take seriously the notion that the order in which mathematics is taught makes a difference, they are poised to rethink much of what happens in mathematics classes. Consider the two algorithms for multiplication of multidigit numbers, displayed in Figure 1. The first one, usually taught in the United States,[1] can be learned without much place value knowledge, whereas using the second requires students to focus on place value ideas. The first algorithm may be easier for students to learn, but the second algorithm is more likely to be learned in connection with the underlying place value ideas, and, as such, it may provide students with a deeper, more connected understanding. Thus, even if educators believe that students need to learn the first algorithm, to promote sensemaking they could start with the second algorithm and teach the first only after students have come to understand the origins and meanings of the partial products in the second.

Consider the sequencing of a typical mathematics lesson. When teaching mathematics, teachers generally tell the students the new concept, show them a procedure to use, and then set the students loose. Could teachers not, at least on occasion, give students a problem or task and *get out of the children's way* as they think about the task and how they want to solve it? Could teachers not ask students to share their different approaches and attempt to develop a climate in class whereby what students say mathematically is valued by all? Could they expect students to share their solutions and to listen to one another? I realize that these ideas seem foreign to some teachers, and I am not asking that they do this every day, but could they then do it a couple of times every unit? Could they begin each unit by posing these open-ended questions?

In each weekly mathematics lesson I teach in a local public school, I have the children sit in a circle, give them manipulatives and paper and pencil, and pose problems for them to think about. One outcome of this experience is that the children have learned that they are expected to listen not only when the teacher speaks but also when other children speak, and they are beginning to develop more sophisticated listening skills.[2]

For teachers who wish to provide students opportunities to think creatively but do not know where to begin, one approach is to change the usual order of a lesson. Instead of beginning with the explanation and ending with the classwork or homework, try starting a lesson by giving one of the classwork or homework problems and give the students time to think about the problem and share their approaches *before* showing the solution. An argument against this approach is "How are students supposed to know how to do something before they are shown?" This is a valid question in circumstances in which the mathematics being taught is relatively unrelated or completely isolated from what the students know. But when teaching mathematics, one seldom *successfully* teaches concepts or procedures that are unrelated to everything else the students know.

The second question addresses which should come first, the concepts or the procedures? Many of the preservice teachers from Vignette 1 believed that children should learn procedures first. However, the research on children's mathematical thinking paints a picture indicating that children who learn procedures before concepts may not learn the concepts as well as those children who learned the concepts before the procedures. Is there a middle ground whereby children learn concepts and procedures simultaneously? Perhaps, although I fear that when such instruction is attempted, most children pick up that what is *really* important are the procedures, and hence they do not learn the concepts. If I were responsible for teaching fraction concepts to one of the two children in Vignette 2, I would prefer to teach the first grader *even if the fifth grader were "more intelligent,"* because I believe that the fifth grader's previous procedural fraction work has oriented him away from making conceptual sense. That is, in some way, he has been mathematically damaged.

## Methodology

(3) What types of questions do you pose?

There are many reasons to ask questions in a mathematics class, including to keep students on task (Johnny, are you with me?), to find out what students know (What is 4 times 5?), and to provide students an occasion to share their thinking (Who can share how you thought about this?). But there is another purpose for asking questions, one that too often is ignored by mathematics teachers: to get students thinking (Here is a problem. How might you solve it?). Unfortunately, teachers often answer their own questions or expect students to answer the questions immediately (with *the* answer the teachers have in mind), and they are uncomfortable posing problems or asking open-ended questions. Teachers tend to use questions as a means of moving their lessons toward some desired end, and one unintended consequence of this goal-driven lesson is that students may not be required to think creatively or originally.

The questions teachers pose often reflect the underlying goals they hold for instruction. Alba G. Thompson, Randolph A. Philipp, Patrick W. Thompson, and Barbara A. Boyd (1994) suggested that teachers who are guided by a fundamental image of mathematics as the application of calculations and procedures for deriving numerical results will pose *calculational* questions, whereas teachers guided by an image of mathematics as a system of ideas and ways of thinking will pose *conceptual* questions. For example, consider the problem: "Susan drives 240 miles in 5 hours. What is the average speed of her trip?" A question commonly asked by teachers is "How did you solve this?" Unfortunately, this question more often than not directs students to the calculations they performed, so a likely response is: "I divided 240 by 5, and the answer is 48." A question designed to direct students more toward the underlying conceptual ideas would be: "When you divided 240 by 5 and got 48, what is 48 a number of? That is, to what does 48 refer in this situation?" Such a question is designed to direct students to focus on the quantities in this problem and on the relationships among those quantities.[3] Instead of asking, "What did you do?" ask, "What were you thinking?" Instead of asking, "What calculation did you perform?" ask, "What are you trying to find when you do this calculation (in the situation as you currently understand it)?" Explaining how one solved a problem answers the question "*What* did you do?" but explaining how one thought about a problem answers the question "*Why* did you do that?" Both doing and reasoning are important, but when working with children, teachers tend to focus most attention on the doing. I suggest that teachers be helped to focus also on the reasoning.

## Assessment

(4) What understanding of concepts or knowledge of procedures do your students possess *prior* to instruction? Do you assess what your students learn on a daily basis?

Denise S. Mewborn (1999) reported observing a student teacher who reflected on her instruction on the topic of *time*; after teaching about time for four days, the student teacher came to realize that her students had already known what it was that she was trying to teach. How often is this scenario repeated daily in classrooms?

Most teachers are surprised by some of the problems primary-grades children can solve. Carpenter, Ellen Ansell, Franke, Elizabeth Fennema, and Linda Weisbeck (1993) found that most children, by the end of their kindergarten year, could solve a variety of mathematics problems, including addition, subtraction, multiplication, division, and multistep problems. For example, 50 out of 70 kindergartners in May correctly solved the measurement division problem "Tad had 15 guppies. He put 3 guppies in each jar. How many jars did Tad put guppies in?" and 45 out of 70 correctly solved the multistep problem "Maggie had 3 packages of cupcakes. There were 4 cupcakes in each package. She ate 5 cupcakes. How many are left?" (pp. 434-435). Children use various strategies to solve such problems, and their strategies reflect different levels of sophistication. For example, some will solve the problem "Kris has 5 apples. How many more apples does she need to buy to have 11 altogether?" by putting out 5 counters, then putting more out and counting "6, 7, 8, 9, 10, 11," and then counting the additional counters they put out to see that the answer is 6. Another child will say, "Five (pause), 6, 7, 8, 9, 10 11 (raising fingers as he counts)" and count the six fingers put up. Another child might say, "Five and 5 is 10, and 1 more is 11, so the answer is 6." There are teaching implications for these findings; in particular, one must wonder what happens to a child who needs to model these problems when no counters are available for him or her to use? Most primary-grades children who can solve the division problem like that presented above do so by modeling the task. If they do not have permission to use counters or if the counters are not accessible, most children will be unable to make progress on the problem. Another teaching implication has to do with teachers' expectations. If teachers do not believe that young children are capable of solving such problems, they will not consider posing them.

In my mathematics methods course my students interview children of different ages; they find that whereas primary-grade children understand much *more* than the teachers had expected, intermediate-grade children understand much *less* than they had expected. Evidently years of learning procedures without meaning have left many intermediate-grade children without the kind of understanding needed to build upon. Consider the fifth-grade boy from Vignette 2 who thought 1/3 + 1/3 =

2/6. How could he make sense of a lesson designed to help him understand fraction multiplication?

Only by conducting daily assessment can a teacher determine whether students need what is being taught and whether they are learning what they need to learn. How can a teacher build assessment into each daily lesson? What questions could be posed during the lesson that might clarify what students know? (In addition, *what* is being assessed? Are students expected to learn just procedures and algorithms, or is understanding also valued?)

Understanding develops in a complex and fragile manner, and no two students learn at the same pace because no two students enter with the same knowledge or interest in the subject. One consequence of the complexity and fragility with which understanding develops is that teachers cannot assume that students have learned simply because they have been told. Alba Thompson once said that there is a difference between putting the students through the curriculum and putting the curriculum through the students. Teachers can determine which they are doing only by assessing their students.

## Final Comments

In this article I have provided four questions supervisors might find useful when considering mathematics instruction by elementary school preservice teachers. My emphasis on content reflects my concern that as long as student teachers are guided by an image of mathematics as a set of procedures they are unlikely to value changing how they teach or what they assess.

Mathematics educators continue to believe that basic facts are critically important. However, when and how these basic facts are to be learned is an open question. The *California Mathematics Academic Content Standards* (California Department of Education [CDE], 1998) were written as a reaction to the overstated view that mathematics education reformers have been calling for the elimination of memorization and of the teaching of rules. The new *Standards* are more traditional than the 1992 California *Framework* (CDE), and they call for students to memorize facts and procedures earlier. However, the *Standards* also call for students to develop understanding of mathematical concepts.

Providing the kind of instruction described in this article requires courage because most teachers have been taught traditional mathematics through traditional instruction. That is, the institution called *school*—with support from the recently published *California Mathematics Academic Content Standards* (CDE, 1998), which may be interpreted by teachers, administrators, and curriculum developers as devaluing a deeper understanding of mathematics in favor of memorizing facts and procedures—may provide compelling reasons for teachers *not* to question or alter their current practices. Furthermore, most teachers have not had opportunities to develop deep conceptual understanding of the mathematics they are teaching.

Consequently, if teachers give problems to their students and allow students to use their own solution techniques, the teachers will at times find that their students are better mathematical thinkers than they are. Exposing themselves to this vulnerability may be more than most new teachers are ready to give.

Finally, the important information is in the details. It is easy to say that one values *conceptual understanding*, but the meaning of this term becomes clear to others only through examples and discussion. When student teachers say that they are teaching for understanding, ask for specific examples. Asking students to be more explicit not only will help you determine how the teachers are thinking but may have the more important benefit of helping the teachers step back and look at their own knowledge and beliefs about mathematics and mathematics teaching.

## Notes

1. Many teachers are unaware that different cultures use different algorithms and that no algorithm is *the* way to work a problem, but instead each is *a* way (Philipp, 1996).
2. I do not want to understate the difficulty I have had trying to encourage students to listen to one another. Often I was satisfied if the students simply *looked* at the child who was sharing his or her thinking. Perhaps part of the difficulty came about because I presented lessons only once each week, and the skill of listening to other students as they describe their thinking processes and analyzing these processes is clearly nontrivial. This statement should not be surprising given that adults seem to experience difficulty listening to one another. Someone once said, "No one really listens to anyone else, and if you try it for a while you'll see why."
3. Quantities are distinguished from values; whereas distance, time, average speed, instantaneous speed, and acceleration are quantities, their values are the numbers associated with each. Most classroom discourse is about the values instead of the quantities, so that students may recognize that to solve this problem they divide 240 by 5, but they may not recognize that speed may be conceptualized as the mutual accrual of distance and time.

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